Heterotic Strings on $T^3/\mathbb{Z}_2$, Nikulin Involutions & M-theory

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Based on: B. Acharya, G. Aldazabal, A. Font, K. Narain, IGZ, 2205.09764

Non-supersymmetric compactifications of superstrings are typically unstable, but they could still give new insights on properties of theories that include gravity at quantum level.

The goal is to explore a class of compactifications of heterotic string on $T^3/\mathbb{Z}_2$ in which SUSY is broken and to describe them at the string worldsheet level.

M-theory/Heterotic duality

Heterotic theory with gauge group $E_8 \times E_8$ compactified on T^3: 7d theory has 16 supercharges, momentum lattice is an even self-dual lattice $\Gamma_{(19,3)}$. [Narain]

M-theory on K3: has 16 supercharges and gauge group of rank 22. Membranes wrapped on 2-cycles of K3 are charged under gauge fields with charge lattice $\Gamma_{(19,3)}$. [Hull,Townsend; Witten]

Second cohomology group of K3, with the intersection form of K3, is isometric to $\Gamma_{(19,3)}$.

M-theory/Heterotic duality: New

Non-supersymmetric $\mathbb{Z}_2$ orbifolds of heterotic theory on T^3: a reflection of s of 19 left-moving (bosonic string) directions and 2 of 3 right-moving (superstring) directions of momentum lattice $\Gamma_{(19,3)}$.

Duality suggests compactifications of M-theory on $\mathbb{Z}_2$ orbifolds of K3 surfaces that act similarly on $\Gamma_{(19,3)}$ (reflecting s left- & 2 right-moving directions).

Such involutions of K3 surfaces have been classified.

[Nikulin]

Nikulin involutions

Non-symplectic involution, θ, that acts by (-1) on the holomorphic 2-form but leaves a Kähler form invariant.

K3 quotients are classified in terms of I, the sublattice of $\Gamma_{(19,3)}$ left invariant under θ.

Nikulin involutions

I has rank r ($r := 20 - s$), signature $(r-1,1) = (19-s,1)$, and satisfies $I^*/I = \mathbb{Z}_2^a$.

I is uniquely specified (up to isomorphisms) by three invariants (r, a, δ), where

$$\delta = \begin{cases} 0 & \text{if } P_I^2 \in \mathbb{Z} \ \ \forall P_I \in I^* \\ 1 & \text{otherwise} \end{cases}$$

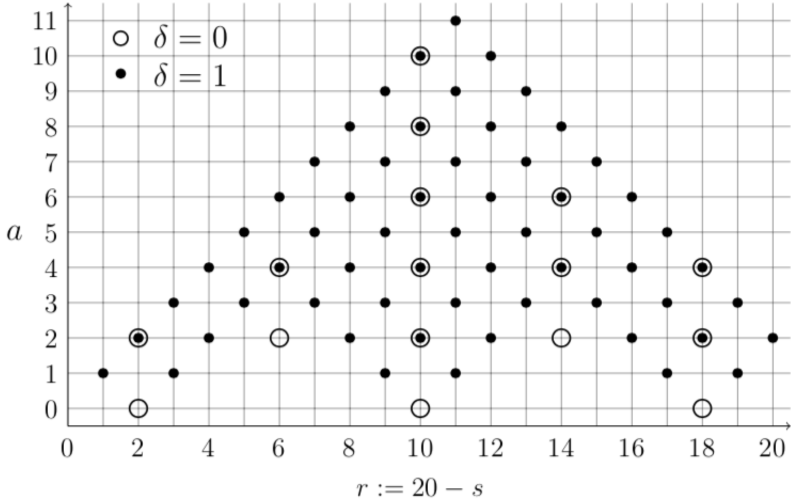
Nikulin involutions

l has rank r ($r := 20 - s$), signature $(r-1, 1) = (19 - s, 1)$, and satisfies $l^*/l = \mathbb{Z}_2^a$.

l is uniquely specified (up to isomorphisms) by three invariants (r, a, δ), where

$$\delta = \begin{cases} 0 & \text{if } P_l^2 \in \mathbb{Z} \ \ \forall P_l \in l^* \\ 1 & \text{otherwise} \end{cases}$$

Λ	U	U(2)	$A_1(-1)$	A_1	E_7	E_8	$E_8(2)$	D_{4m}	D_{4m+2}
(r, a, δ)	(2,0,0)	(2,2,0)	(1,1,1)	(1,1,1)	(7,1,1)	(8,0,0)	(8,8,0)	($4m$,2,0)	($4m+2$,2,1)



(r, a, δ) determine all 75 invariant lattices (I) of signature $(r-1, 1)$ embedded primitively in K3 lattice $\Gamma_{(19,3)}$.

[Nikulin]

Main results

Give an exact worldsheet description of heterotic strings on $T^3/\mathbb{Z}_2$ using the formalism of asymmetric orbifolds.

Characterise flows in the moduli space of heterotic orbifold theory: this yields transitions which connect models with different (r, a, δ).

- Supergravity limit
- Worldsheet theory
- Remarks

Flat connections on $T^3/\mathbb{Z}_2$

Study low-energy field theory to specify the moduli space and to define the orbifold action on gauge degrees of freedom. Specify flat gauge connections on heterotic $E_8 \times E_8$ or $Spin(32)/\mathbb{Z}_2$ gauge bundle.

In Euclidean coordinates (x_1, x_2, x_3) describe generators of the fundamental group of $T^3/\mathbb{Z}_2$ as 3 commuting translations of T^3, g_1, g_2, g_3, and the orbifold generator g_θ which is order two on T^3:

$$g_i : \; x_i \to x_i + 1 \,, \quad g_\theta : \; (x_1, x_2, x_3) \to (-x_1, -x_2, x_3 + \tfrac{1}{2})$$

Flat connections on $T^3/\mathbb{Z}_2$

Fundamental group of $T^3/\mathbb{Z}_2$ can be described by

$$
\begin{array}{rcl}
g_i g_j = g_j g_i \ , & & \forall i, j = 1, 2, 3 \\
g_\theta g_1 g_\theta^{-1} = g_1^{-1} \ , & & g_\theta g_2 g_\theta^{-1} = g_2^{-1} \\
g_\theta g_3 g_\theta^{-1} = g_3 \ , & & g_\theta^2 = g_3
\end{array}
$$

A flat connection on the heterotic group $E_8 \times E_8$ or $Spin(32)/\mathbb{Z}_2$ is specified by a set of four Wilson lines, one for each generator, obeying these relations. There exist different families of solutions.

Higgs branch solutions

Let us consider the flat connection to be restricted to an $SU(2)$ subgroup of the gauge group:

$$g_1 = e^{i\phi_1\sigma_3},\ g_2 = e^{i\phi_2\sigma_3},\ g_\theta = i\sigma_2,\ g_3 = -\mathbb{1}\ \ (g_\theta^4 = \mathbb{1})$$

The low energy field theory contains two light scalars, which naturally form a complex scalar field.

At the origin of moduli space ($\phi_{1,2} = 0$) there is an $SO(2)$ subgroup of $SU(2)$ which commutes with the flat connection.

The 7d theory has an enhanced $SO(2)$ gauge symmetry at the origin, broken for generic values of $\phi_{1,2}$.

Coulomb branch solutions

Identity connected solutions:

$$g_1 = g_2 = \mathbb{1}, \;\; g_\theta = e^{i\phi_3 \sigma_3}, \;\; g_3 = g_\theta^2$$

These solutions generically break the gauge symmetry down to the maximal torus, $U(1)$, and have one modulus.

We refer to these solutions as Coulomb branch vacua.

Higgs & Coulomb branch solutions

At the origin of Higgs branch solution ($\phi_1 = \phi_2 = 0$), solution is equivalent to a particular Coulomb branch solution: the two types of branches of moduli space intersect there.

Moduli of either branch could be switched on at the intersection point: a transition between branches.

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Asymmetric orbifolds

Orbifold group element g reflects s left-moving (b) and 2 right-moving (f) directions: *asymmetric orbifolds*. [Narain, Sarmadi, Vafa]

g^2 acts by (-1) on space-time fermions: the orbifold group becomes $\mathbb{Z}_4$.

Asymmetric orbifolds

Action on $P \in \Gamma_{(19,3)}$, $P = (P_N, P_I)$:

$$g : |P_N, P_I\rangle \to f(P_N)\, e^{2\pi i\, P_I . v} |-P_N, P_I\rangle$$

$4v \in I$: $g^4 |P_N, P_I\rangle = |P_N, P_I\rangle$

$\mathbb{Z}_4$ phase: $f(P_N) f(-P_N) = e^{2\pi i\, P_N^2}$

1-loop partition function:

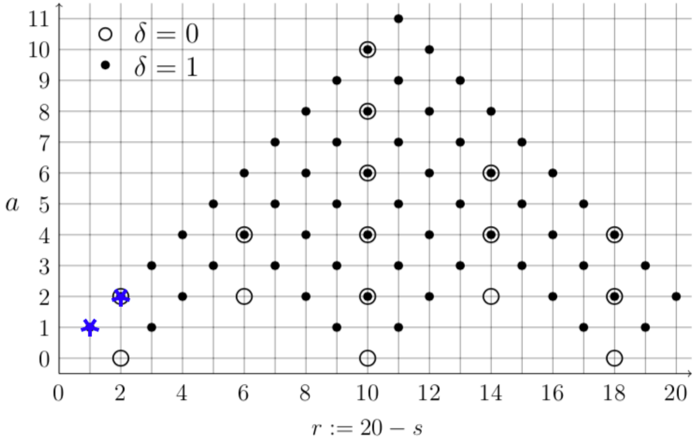
$$Z = \frac{1}{4} \sum_{m=0}^{3} \sum_{n=0}^{3} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Z_{m,n}$$

Level matching codition:

$$2v^2 + \frac{s}{4} \in \mathbb{Z}$$

Asymmetric heterotic orbifolds can be realised for all triples (r, a, δ) except for 2 points: $(1, 1, 1)$, $(2, 2, 1)$.

I is small and there is no solution for v that satisfies level matching condition for these points.



Spectrum

Tachyons do not appear in the untwisted sector. They generically appear in the twisted sectors in some regions of moduli space, *e.g.* for values of the circle radius $R_{min} < R < R_{max}$. They become massless at the endpoints and massive outside this interval.

At one loop level an effective potential might be generated which drives the theory to regions where tachyons appear.

[Acharya, Aldazabal, Andrés, Font, Narain, IGZ]

s-transitions

g acts by reflecting s left- & 2 right-moving directions. In $(1, g)$ sector:

$$Z_{0,1} = \underbrace{\hat{Z}_{0,1}}_{\substack{I\text{, fermions} \\ \text{non-compact bosons}}} \underbrace{\left(\frac{2\eta}{\vartheta_2}\right)^{\frac{s}{2}} \left(\frac{2\bar{\eta}}{\bar{\vartheta}_2}\right)}_{N}$$

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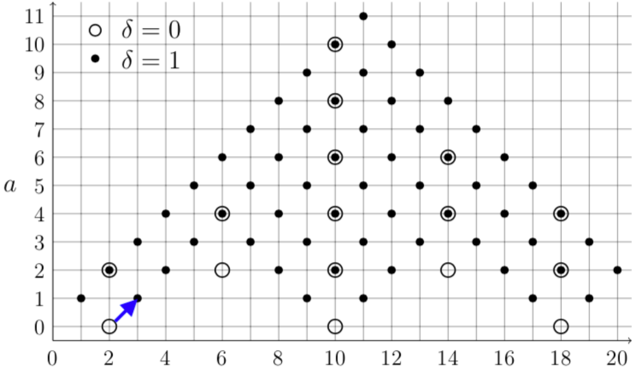
$$\left(\frac{2\eta}{\vartheta_2}\right)^{\frac{1}{2}} = \frac{1}{q^{\frac{1}{24}}\prod_n(1+q^n)} = \frac{1}{\eta}\sum_n q^{n^2}e^{i\pi n} = \frac{1}{\eta}\sum_{P\in A_1} q^{\frac{1}{2}P^2}e^{2\pi i P\cdot v_1}$$

where $P \in A_1$ *i.e.* $P = \sqrt{2}n$ and $v_1 = (v_{1L}; v_{1R}) = (\frac{1}{2\sqrt{2}}; 0)$.

$$Z_{0,1} = \hat{Z}_{0,1}\left(\frac{2\eta}{\vartheta_2}\right)^{\frac{s}{2}}\left(\frac{2\bar{\eta}}{\bar{\vartheta}_2}\right) = \hat{Z}_{0,1}\left(\frac{2\eta}{\vartheta_2}\right)^{\frac{s-1}{2}}\left(\frac{2\bar{\eta}}{\bar{\vartheta}_2}\right)\frac{1}{\eta}\sum_{P\in A_1} q^{\frac{1}{2}P^2} e^{2\pi i P\cdot v_1}$$

Decrease of s by 1 is accompanied by the emergence of a lattice sum over A_1. This lattice sum can be absorbed in the contribution of invariant lattice thereby increasing r by 1. Equality holds in all sectors.



Moduli

g acts on the right-moving $\mathbb{T}^2 \times S^1$ as rotation on $\mathbb{T}^2$: $\tilde{X}_1$ and $\tilde{X}_2$ are directions in N and $\tilde{X}_3$ in I.

Consider the $(3, 1, 1)$ model with $I = U + A_1$. A_1 can be realized by a left-moving boson Y. The Kac-Moody currents are $J_3 = \partial Y$ and $J_\pm = e^{\pm i\sqrt{2}Y}$.

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g acts as $Y \to Y + 2\pi v_1$, where $v_1 = (\frac{1}{2\sqrt{2}}, 0)$: $J_3 \to J_3$ and $J_\pm \to -J_\pm$.

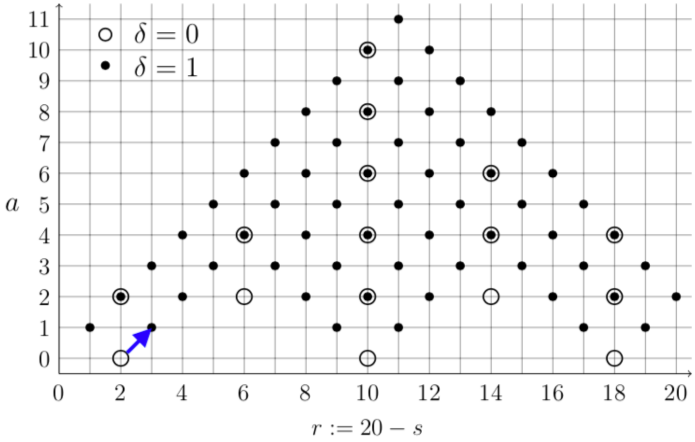
Exactly marginal operators: $J_3 \partial\tilde{X}_3$, $J_\pm \partial\tilde{X}_1$, $J_\pm \partial\tilde{X}_2$.

Moduli

If we deform by giving a vev to $J_3\partial\tilde{X}_3$ then $J_\pm\partial\tilde{X}_{1,2}$ become massive. This deformation is along the Coulomb branch because it leaves the $U(1)$ gauge symmetry unbroken. This is all part of the $(3,1,1)$ moduli space.

If we give a vev to say $(J_+ + J_-)\partial\tilde{X}_1$ then the only invariant state which remains massless is $(J_+ + J_-)\partial\tilde{X}_2$. In this branch, called Higgs branch, there are two moduli. The $U(1)$ gauge symmetry is broken. This is part of the $(2,0,0)$ moduli space.

Starting from a consistent orbifold model for a given s and going to a point with $SU(2)$ enhancement, we can move to a different branch of the moduli space with s shifted by 1.



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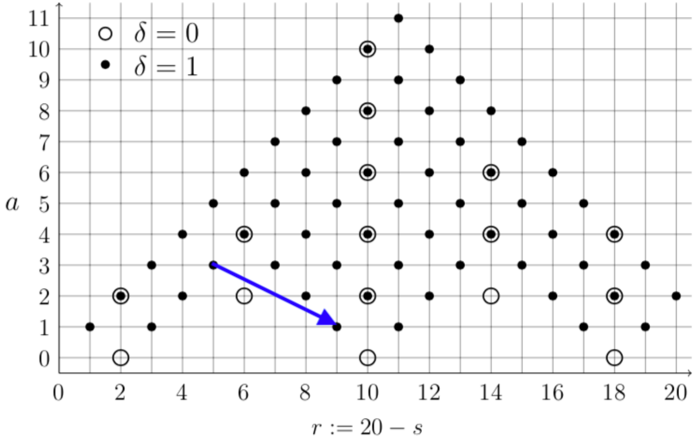
All heterotic models associated with triples (r, a, δ) are connected through s-transitions, *i.e.* through moduli aquiring vevs.

In M-theory there are membranes wrapping the cycles of the K3 surface. Transitions are non-perturbative and happen when a membrane state becomes massless and acquires a vev.

Does a big unique moduli space exist where each (r, a, δ) form a subspace of it?

Maybe! There are models for which not *all* the shift vectors can be connected through automorphisms of the lattice.



• Heterotic orbifolds are characterised by (r, a, δ) together with the shift vector v. In M-theory this implies that the involution acts by some phases on the membrane states. It would be interesting to understand the origin of these phases in M-theory.

• It would be interesting to construct T^3 heterotic orbifolds corresponding to other non-symplectic (higher order) automorphisms of $\Gamma_{(19,3)}$, as well as their higher dimensional counterparts T^d and $\Gamma_{(16+d,d)}$.

Thank You!

Asymmetric orbifolds

$\mathbb{Z}_4$ phase: $f(P_N)f(-P_N) = \begin{cases} 1 & \text{if } P_N^2 \in \mathbb{Z} \quad \forall P_N \in N^* \\ -1 & \text{otherwise} \end{cases}$

$$g^2 : |P_N, P_I\rangle \to f(P_N)f(-P_N)\, e^{4\pi i\, P_I . v}|P_N, P_I\rangle$$

with $f(P_N)f(-P_N) = e^{2\pi i\, P_N^2} = e^{2\pi i\, P_I^2}$ $(P_N^2 + P_I^2 = P^2 \in 2\mathbb{Z})$.

This can be written as $e^{2\pi i\, P_I^2} = e^{2\pi i\, P_I \cdot w}$, $\forall P_I \in I^*$ where $w \in I^*/I$.

1-loop partition function:

$$Z = \frac{1}{4} \sum_{m=0}^{3} \sum_{n=0}^{3} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Z_{m,n} \ , \quad Z_{m,n} = \mathrm{tr}_{\mathcal{H}_m} g^n q^{L_0} \bar{q}^{\bar{L}_0}$$

Level matching codition:

$$2v^2 + \frac{s}{4} \in \mathbb{Z}$$

Consistent operator interpretation in g^2-twisted sector (*i.e.* consistent action of g on $\mathcal{H}_2$):

$$w^2 + \frac{s-2}{2} \in 2\mathbb{Z}$$

Nikulin involutions

I has rank r ($r := 20 - s$), signature $(r-1,1) = (19-s,1)$, and satisfies $I^*/I = \mathbb{Z}_2^a$.

I is uniquely specified (up to isomorphisms) by three invariants (r, a, δ), where

$$\delta = \begin{cases} 0 & \text{if } P_I^2 \in \mathbb{Z} \;\; \forall P_I \in I^* \\ 1 & \text{otherwise} \end{cases}$$

Define the normal lattice $N := I^\perp \cap \Gamma_{(19,3)}$ with rank $s+2$, signature $(s,2) = (20-r,2)$, $N^*/N = I^*/I = \mathbb{Z}_2^a$.

N is also uniquely determined by the triple (r, a, δ).